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Abstract

This abstract considers a combine Location/Routing Problem. We define an auxiliary network and give a compact formulation of the problem in terms of finding a set of paths in the auxiliary network that fulfill additional constraints. The LP solution to the considered model provides an initial lower bound and is also used in a rounding procedure that provides the initial solution for a Tabu Search heuristic. Additionally, we propose a different lower bound that improves noticeably the value of the original LP bound. The results of computational testing on a set of randomly generated instances are promising.

Key words : location, routing, location/routing, tabu search.

Résumé

Cet article considère un problème combiné de localisation et de tournées de véhicules. On définit un réseau auxiliaire et on présente une formulation compacte du problème consistant à trouver une famille de chemins sur le réseau auxiliaire qui satisfait à quelques contraintes additionnelles. La solution à la relaxation linéaire du problème sur le réseau donne une borne inférieure et est aussi utilisée dans une heuristique d'arrondissement qui produit une solution initiale pour une heuristique de recherche avec tabous. De plus, on propose une autre borne inférieure qui améliore notablement la valeur de celle fournie par la relaxation linéaire. Les résultats des expériences de calcul réalisées sur un ensemble de problèmes générés aléatoirement sont très encourageants.

Mots-clefs : localisation, tournées de véhicules, localisation/tournées, recherche avec tabou.

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A compact model and tight bounds for a combined Location\Routing Problem

1 Introduction

The design of logistic systems requires a number of different types of strategic decisions. One of the higher level decisions that must be addressed involves the location of the facilities from which the activity of the system will be managed. Facilities must be located (possibly in more than one layer) so as to minimize the sum of the operating costs of the system. These costs include not only the set-up costs of the facilities that are to be located and, in some cases, the distribution costs between facilities, but also the costs incurred for servicing the demand of the clients. In some situations, service is given at the location sites so that clients have to visit the facilities. This is, for instance, the case when facilities are hospitals, schools, post-offices, etc.

Here the operational level requires decisions concerning the allocation of clients within the set of open plants, and it gives rise to the location\allocation problems that have been widely studied for a large variety of settings and conditions (see, for instance, Mirchandani and Francis 1990). In general, location\allocation problems take into account allocation costs of clients to plants as well as other characteristics of the systems such as the capacities of the open plants and the demands of the clients.

In other situations, however, service is given at the clients' sites. Now, the operational level requires making additional decisions concerning the management of the fleets, giving rise to location\routing (LR) problems. In addition to the previously mentioned characteristics, LR problems take into account the distances between clients as well as other characteristics of the systems such as the number of available vehicles, their capacities, etc.

In many practical applications, the location of facilities not only affects locational costs, but also has a major impact on routing costs (Webb 1968; Salhi and Rand 1989). In these situations the overall location\routing problem is, in fact, a multilevel problem that should be addressed jointly, so a combined LR model is appropriate. Nevertheless, in practice, due to the difficulty of both location and routing problems, the two problems have been very often solved

independently. In the last years, however, several works have addressed combined LR problems of various characteristics. Yet, as the model becomes more general, the solution methodology that is applied often resorts to iterative heuristic approaches where at each iteration a location problem plus a routing problem are solved independently.

There are many different aspects that characterize the various types of LR problems. Laporte (1988) reviews examples, applications and algorithms for deterministic LR problems. A more recent survey where stochastic models are also considered is Min, Jayaraman, and Srivastava (1998). In this work we focus on a deterministic, static (one single period) problem. In general terms, deterministic, static LR problems differ from each other in:

- the type of facilities to be located: *primary* if they are the origins and destinations of vehicle journeys, *secondary* when they can only be intermediate depots;
- the number of facilities to be located and whether they have capacities or not;
- the number of available vehicles and whether they have capacities or not.

Exact algorithms for LR problems with a single primary facility have been proposed for uncapacitated vehicles (Ghosh 1981; Stowers and Palekar 1993; Averbakh and Berman 1994). A heuristic algorithm for the case of capacitated vehicles is presented in Chien (1993). The capacitated case for locating a single secondary location has also been studied in Laporte and Nobert (1981) where an exact algorithm is proposed.

The case of multiple facilities with uncapacitated vehicles, has also been considered in the literature. Primary locations have been addressed heuristically (Srivastava 1993) and with exact methods (Zografos and Samara 1989; List and Mirchandani 1991; ReVelle, Cohon, and Shobrys 1991). Exact algorithms for secondary facilities have been proposed by Laporte, Nobert, and Pelletier (1983) and by Laporte and Dejax (1989).

LR problems with multiple facilities and capacitated vehicles are more complex. We are only aware of one work (Nambiar, Gelders, and Van Wassenhove 1981) where primary locations have been considered. Secondary facilities have been addressed heuristically both in the uncapacitated case (Gillett and Johnson 1976; Jacobsen and Madsen 1980; Srivastava and Benton 1990) and when facilities have capacities (Perl and Daskin 1984; Perl and Daskin 1985; Bookbinder and Reece 1988). To the best of our knowledge problems with capacitated secondary facilities have only been solved exactly in Laporte, Nobert, and Arpin (1986) and in Laporte, Nobert, and Taillefer (1988).

The LR problem that we study in this work considers capacitated primary facilities and one single vehicle associated with each open facility. We want to find a set of plants to open and a set of routes to service clients (one from each open plant) so as to minimize the sum of the set-up costs plus the routing costs. We define an auxiliary network that allows formulating the problem in terms of finding a set of paths in the auxiliary network, one associated with each possible location, that fulfill some additional constraints.

We apply a solution method that uses the LP solution to our model to obtain a first lower bound and as starting point of a Tabu Search (TS) heuristic. The initial solution to the heuristic

is obtained by applying a rounding procedure to the LP solution to the model. The Tabu Search heuristic consists of a series of intensification/diversification iterations. The diversification phase operates mainly at the location level and selects new subsets of open facilities. The intensification phase is a local search phase that is applied to the current routing subproblem. Feasibility is not easy to ensure in the intensification phase, due to the capacity constraints. For this reason we use a strategic oscillation scheme that allows violation of feasibility and considers a modified objective function that includes a penalty term associated with infeasible solutions. The weight given to such a penalty is dynamically updated based on the history of the search as in Díaz and Fernández (2001).

Additionally, we propose a lower bound that generally provides a major improvement with respect to the LP bound of the proposed model that, in general, is very week. The new lower bound has two terms that are derived from the structure of each of the two main components of the original problem: The first refers mainly to the location costs and the second to the routing costs. The term of the location costs is obtained by solving an *ad hoc* Knapsack Problem whereas the term of the routing costs is obtained by solving an *ad hoc* Asymmetric Traveling Salesman Problem.

The quality of the upper and lower bounds has been tested in a series of computational experiments. The results show the good quality of both the new lower bound and the Tabu Search heuristic. The required times are small for problems of this difficulty especially, taking into account that most of the test problems could not be optimally solved by CPLEX 6.5 within 48 hours of computation time.

The paper is organized as follows. In Section 2 we define the problem and we propose a model for the problem defined in terms of an auxiliary network. Section 3 describes the rounding procedure and the Tabu Search heuristic. In Section 4 we show how to obtain a new lower bound to the problem. The details of the computational experiments as well as the obtained results are presented in Section 5. We end the paper with some conclusions and final remarks.

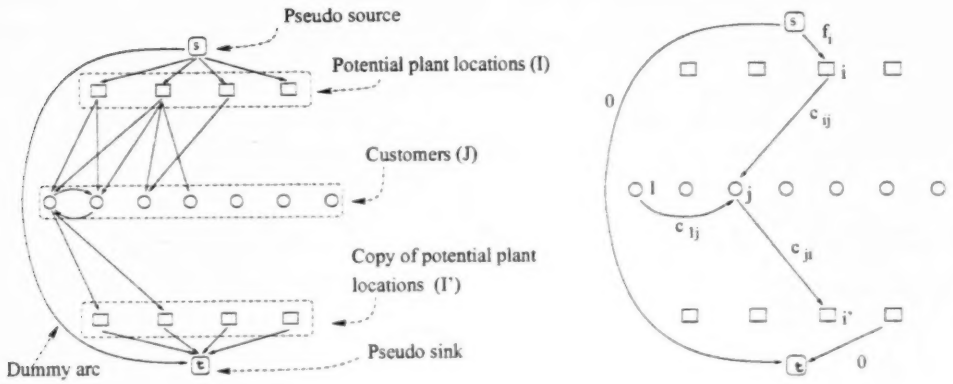
2 The model

Let I denote the set of indices of potential locations for the plants and J the set of indices for the clients. For each $i \in I$ let f_i be the cost for opening the plant at site i and let v_i be the capacity of plant i. For each $j \in J$, d_j denotes the demand of client j. Also let $C = (c_{ij})$, $i, j \in I \cup J$ be the matrix that contains the travel costs from clients to plants, and between clients. We want to find a set of plants to be opened, and a set of routes to service clients from the open plants such that the opening costs plus the cost of the routes is minimized. The cost of a route is defined to be the sum of the costs of each travel in the route. We assume that *i)* only one route takes place from each open plant; *ii)* the routes start and end at the same plant; *iii)* the capacity of an open plant, also represents the capacity of the vehicle for the route associated to the plant; and *iv)* the demand of each client must be satisfied from one single plant; that is, each client is visited by exactly one route.

To formulate the problem in terms of a network problem with additional constraints we define the network $N = (V, A)$. The set of nodes is defined to be $V = \{s, t\} \cup I \cup I' \cup J$, where

I' is a copy of set I, and s and t are, respectively, a pseudo source node and a pseudo sink node (see Figure 1a). The set of arcs is

$$A = \{(s,i)|\ i \in I\} \cup \{(i,j)|\ i \in I, j \in J\} \cup \{(j_1,j_2)|\ j_1,j_2 \in J; j_1 \neq j_2\} \cup$$
$$\{(j,i')|\ j \in J, i' \in I'\} \cup \{(i',t)|\ i' \in I'\} \cup \{(s,t)\}$$



a) Nodes and arcs

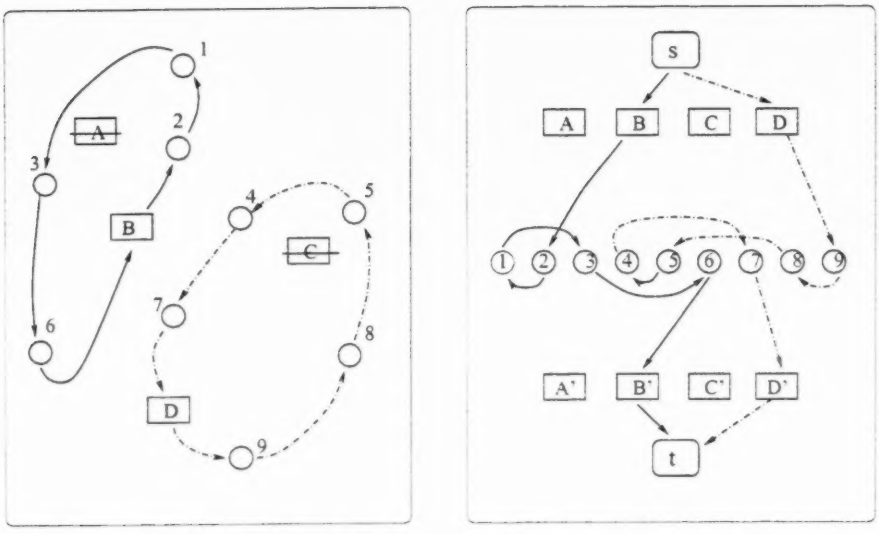
b) Arc costs

Figure 1: Auxiliar network N(V,A)

The cost function $\tilde{c}$ (see Figure 1b) on the arcs of N is given by:

- $\tilde{c}_{si} = f_i, \forall i \in I$;
- $\tilde{c}_{ij} = \tilde{c}_{ji'} = c_{ij}, \forall i \in I, j \in J$ (i' denotes the copy of node i);
- $\tilde{c}_{j_1 j_2} = c_{j_1 j_2}, \forall j_1, j_2 \in J, j_1 \neq j_2$;
- $\tilde{c}_{i't} = 0, \forall i' \in I'$;
- $\tilde{c}_{st} = 0$.

We next explain how to use the above network to find a set of paths, one associated with each potential plant, that will represent the set of plants to be opened as well as the routes to service clients from open plants. Consider any path P from s to t in N. If $P = \{(s,t)\}$, P is called *trivial path* whereas if P does not contain the arc (s,t), it is a *non-trivial* path. Let i_s and i'_t denote the indices of the plants connected with s and t respectively in a non-trivial path P. P can be seen as a route starting at i_s and ending at i'_t. When $i_s = i'_t$, the route starts and ends at the same plant. By definition of the cost function $\tilde{c}$, the cost of P is the sum of the opening cost for plant i_s plus the overall cost of the route. On the other hand, trivial paths are related to not opening a plant and, thus, they have cost 0. The equivalence between solutions to the LR problem and paths in the defined network can be further appreciated in Figure 2.



a) Original solution

b) Equivalent solution in $N(V, A)$

Figure 2: Typical solution in both schemes

Therefore, the combined LR problem can be modeled as the problem of finding a set of paths in the network that fulfill some additional constraints. Specifically, for $v \in V$, let $A(v)^+ = \{a \in A : a = (v, u)\}$ and $A(v)^- = \{a \in A : a = (u, v)\}$ and for $S \subseteq I$ let $A(S) = \{a \in A : a = (u, v), u, v \in S\}$. We define the decision variables $x_a^k, a \in A, k \in I$ to be 1 if arc a is used in route k and 0 otherwise. Then, the problem can be modeled as:

$$\text{(LR1)} \quad \min \sum_{k\in I}\sum_{a\in A} \tilde{c}_a x_a^k \tag{1}$$

$$\sum_{k\in I}\sum_{a\in A(s)^+} x_a^k = |I| \tag{2}$$

$$\sum_{a\in A(v)^+} x_a^k = \sum_{a\in A(v)^-} x_a^k \qquad \forall v \in V\backslash\{s,t\}, \forall k \in I \tag{3}$$

$$\sum_{k\in I}\sum_{a\in A(t)^-} x_a^k = |I| \tag{4}$$

$$x^k_{(s,i)} = x^k_{(i',t)} \qquad \forall i,k \in I \tag{5}$$

$$\sum_{j\in J}\sum_{a\in A(j)^+} d_j x_a^k \leqslant \sum_{i\in I} v_i x^k_{(s,i)} \qquad \forall k \in I \tag{6}$$

$$\sum_{k\in I}\sum_{a\in A(j)^-} x_a^k = 1 \qquad \forall j \in J \tag{7}$$

$$\sum_{k\in I}\sum_{a\in A(S)} x_a^k \leqslant |S| - \left\lceil \frac{\sum_{j\in S} d_j}{v_{max}} \right\rceil \qquad \forall S \subseteq J \tag{8}$$

$$x^k_{(s,i)} = 0 \qquad \forall i \neq k; i,k \in I \tag{9}$$

$$x_a^k \in \{0,1\} \qquad \forall a \in A, k \in I. \tag{10}$$

Constraints (2)-(4) are the flow conservation equations that guarantee that the solution defines $|I|$ s-t-Paths. Equations (5) guarantee that the routes start and end at the same plant while inequalities (6) ensure that the capacity of vehicles is not violated and that no client is served from a plant that is not open. Equalities (7) require that each client is visited by exactly one route. Constraints (8) forbid routes that are not connected with any plant. They are a generalization of the well-known subtour elimination constraints that, additionally, take into account the capacity of the plants. Here v_{max} denotes the maximum capacity of all plants. Constraints (8) are a variation of a class of inequalities that were proposed by Laporte and Nobert (1983) in a somewhat different context of pure routing problems. Note that the aggregation over all plants of the subtour elimination constraints associated with a given set of clients is valid, given that feasible sets of routes are node disjoint. Constraints (9) are used as a means to associate each path with a specific plant, by only permitting the path to go through one of the plants.
LR1 is a linear integer program with $|I| \times (2 \times |I| + 2 \times |I| \times |J| + |J| \times (|J| - 1) + 1)$ binary variables. However, (9) together with constraints (2)-(5) imply that $x^k_{(i',t)} = 0 \quad \forall i' \neq k; i', k \in I$, $x^k_{(i,j)} = 0 \quad \forall i \neq k; i,k \in I; j \in J$, and $x^k_{(j,i')} = 0 \quad \forall i' \neq k; i', k \in I; j \in J$. Therefore, the actual number of variables can be reduced in $2|I| \times (|I| - 1) \times |J|)$.
As usual in routing problems, there is a number of subtour elimination constraints (8) that is exponential in the number of clients. Additionally, the model has a number of constraints of other types that is bounded by $O(|I| \times |J|)$.

Indeed, the LR problem that we consider in this paper can also be modeled without stating the problem in terms of an *ad hoc* network. For instance, the most general three index model of

Perl and Daskin (1985) can be adapted to our problem. In this case the set of decision variables necessarily should include binary variables that indicate if client j is visited immediately after client l in route k, which are a subset of set of the decision variables used in $LR1$. Although this could be avoided, typically one would define additional decision variables to know if a facility is open at site k as well as other decision variables to indicate if client i is visited by route k. The resulting model that does not differ essentially from LR1, can also be easily derived from $LR1$ with a slight transformation.

We use a solution to the LP relaxation of LR1 as a starting point of the heuristic approach that is described in the next section. The LP relaxation of LR1 is strengthened by including the following aggregated demand constraint:

$$\sum_{i \in I} v_i x^i_{(s,i)} \geqslant LC \tag{11}$$

where LC is the solution to the subset sum problem

$$(\text{LC}) = \min \sum_{i \in I} v_i y_i \tag{12}$$

$$\sum_{i \in I} v_i y_i \geqslant D = \sum_{j \in J} d_j \tag{13}$$

$$y_i \in \{0,1\}, \ i \in I. \tag{14}$$

Constraint (11) guarantees that the overall capacity of the plants that are open is enough as to satisfy the overall demand of the clients. Note that (11) is at least as good as the classical aggregated demand constraint where the right hand side is D.

Let RLR1 denote the LP relaxation to model LR1 strengthened with constraint (11) and where the condition that no client is served from a plant that is not open implied by constraints (6) has been expressed in its disaggregated form:

$$x^i_a \leqslant x^i_{(s,i)}, \forall i \in I. \tag{15}$$

When solving RLR1, in order to have a linear programming problem of a reasonable size, initially we drop all constraints (8). We then apply an iterative LP solver scheme where inequalities (8) are added only when they are violated by the obtained continuous solution.

3 Tabu Search

The Tabu Search heuristic (Glover and Laguna 1997) that we apply to obtain solutions for LR1 consists of a series of iterations, each of which performs an intensification phase followed by a diversification phase. Initially we apply a rounding procedure to the optimal solution to RLR1 that provides the first binary solution to which the intensification phase is applied. In subsequent iterations, intensification is applied to the result of the diversification phase. All the solutions

that we consider in the Tabu Search heuristic satisfy all constraints (2)-(10) excepting, possibly, capacity constraints (6). Since solutions that violate some capacity constraint are allowed, the heuristic presents a strategic oscillation behavior.

3.1 Rounding Procedure

The Rounding heuristic is a simple constructive procedure that does the following:

1. Solve RLR1. Let $\bar{x}$ denote an optimal solution.
2. Obtain an integer solution $\hat{x}$ by rounding $\bar{x}$ as follows:

   R_1 Open all plants that are fully or partially used in the solution $\bar{x}$. That is, if $\bar{x}^i_{(s,t)} < 1$ then, open plant i. Let O denote the resulting set of indices of open plants.

   R_2 For each client j,

   i. Evaluate the ratio $\delta^j_i = \left(\sum_{a \in A(j)^-} \bar{x}^i_a \right) / (1 - \bar{x}^i_{(s,t)})$, for each $i \in O$

   Note that $\delta^j_i \leqslant 1$, since δ^j_i represents the ratio between the flow that arrives to client j in the (non-empty) route of plant i, and the actual flow of the route.

   ii. Allocate client j to plant $i(j)$ such that $\delta^j_{i(j)} = \max_{i \in O} \delta^j_i$.

   iii. Apply the nearest insertion criterion to place client j on route $i(j)$.

   R_3 Close any open plant that does not have any client assigned to it if (6) is not violated.

The result of the rounding procedure is a set of open plants and a set of routes, each of them associated with one different open plant. Some of the routes may violate the capacity constraints of their associated plants, since capacity constraints (6) are not taken into account in the rounding procedure. However, by construction, the obtained solution satisfies all other constraints (2)-(10). The Tabu Search (TS) heuristic tries to improve the solutions of the rounding procedure while reducing unfeasibility. To this end i) we add a penalty term to the objective function that weights the violation of capacity constraints, and ii) we explore two simple neighborhoods that modify the assignment of at least one client (this is required to recover feasibility when the solutions violate some capacity constraint). For any solution x, let

$$s(x) = \sum_{i \in I} \max \left\{ \sum_{j \in J} \sum_{a \in A(j)^+} x^i_a - v_i \times x^i_{(s,i)}, 0 \right\} = \sum_{i \in O(x)} \max \left\{ \sum_{j \in J} \sum_{a \in A(j)^+} x^i_a - v_i, 0 \right\} \quad (16)$$

denote its overall violation of capacity constraints. The modified objective function that we consider both in the intensification and the diversification phases is:

$$\sum_{i \in I} \sum_{a \in A} \bar{c}^i_a x^i_a + P \times s(x) = \sum_{i \in O(x)} \sum_{a \in A} \bar{c}^i_a x^i_a + P \times s(x), \quad (17)$$

where P is the weight factor and $O(x)$ denotes the set of indices of open plants for solution x. Note that for feasible solutions the two objectives (1) and (17) give the same value. The penalty weight P is dynamically updated taking into account the history of the search. The strategy

that we use for updating the penalty weight has been previously used by Díaz and Fernández (2001) for the Generalized Assignment Problem. In particular, P is updated according to the expression $P := P \times \alpha^{\beta}$, where α is dynamically updated using the information provided by the medium-term memory, and β is also dynamically updated but using the information of the short-term memory. In our case, $\beta = \frac{infeas}{niter1-1} - 1$, where $infeas$ is the number of infeasible solutions obtained in the last $niter1$ iterations and α is initially set to the value 2 and updated every $niter2$ iterations according to $\alpha := min\{\alpha + 0.01, 3\}$. Note that for a given value of α the value of P varies in $\left[\alpha^{-1} \times P, \alpha^{\frac{1}{niter-1}} \times P\right]$. Thus, the bigger is the value of α, the larger is the range of variation for P. Additionally, since the values of α are always greater than or equal to one, P will only increase when all the solutions found in the last $niter1$ iterations are infeasible.

3.2 The Intensification Phase

The intensification phase is basically oriented to look for good-quality feasible solutions. Since the initial solutions may violate some capacity constraint (6), one of the goals is reducing unfeasibility when it occurs. As mentioned previously, in order to recover feasibility it is necessary to modify the assignment of at least one client. This is achieved by exploring two simple neighborhoods. The first one is the reassignment neighborhood (denoted $N1$), and the second one is the interchange neighborhood (denoted $N2$). For a given solution x, $N1(x)$ contains all the solutions that only differ from x in the assignment of one single client, which is visited by a different route (see Figure 3).

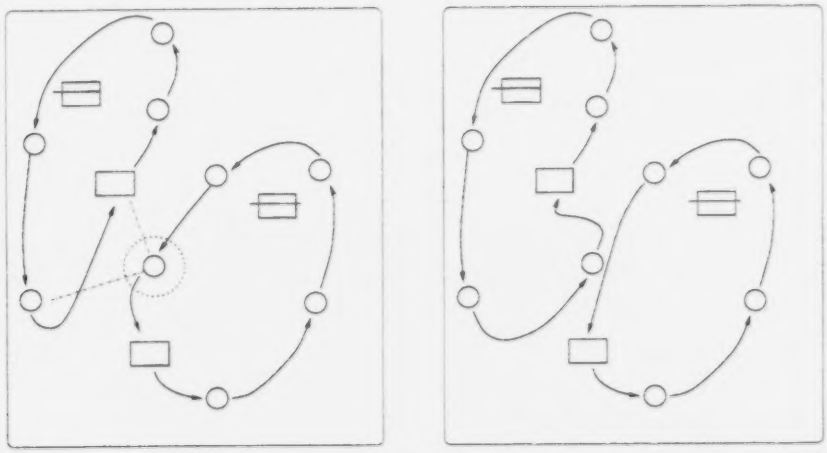


Figure 3: Neighborhood $N1(x)$. Reassignment of one customer

Similarly, $N2(x)$ contains all solutions that only differ from x in the assignment of exactly two clients that interchange the routes that visit them (see Figure 4). Note that, for a given solution x, moves in $N2(x)$ have a fixed number of clients in each route. On the other hand, moves in $N1(x)$ are more restrictive in terms of feasibility. For this reason, we chose to explore $N1(x) \cup N2(x)$, which is a more flexible option. For a given solution x we evaluate all non-tabu moves in $N1(x) \cup N2(x)$ and select the one with a better value with respect to (17). When

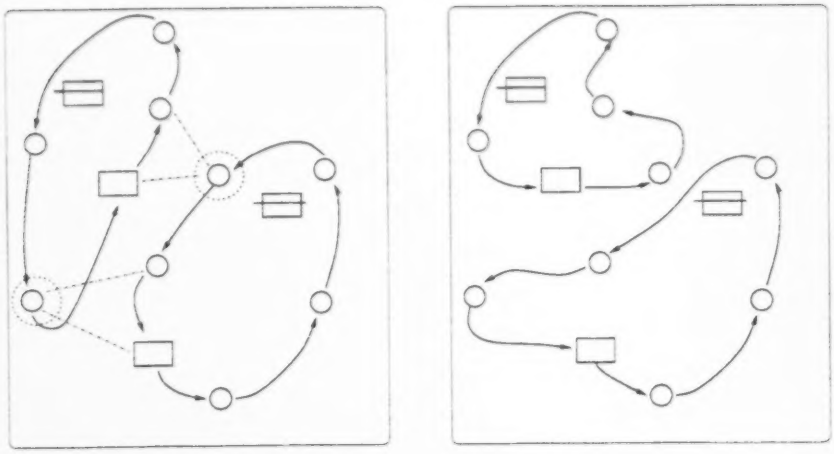


Figure 4: Neighborhood $N2(x)$. Interchange the assignment of two customers

a client j is reassigned from plant $i1$ to plant $i2$ the reverse move is tabu active (and thus, forbidden) for a fixed number of iterations. We apply the aspiration criterion to bypass tabu restrictions if they result in solutions that improve the incumbent. As usual, the incumbent solution is the best feasible solution found so far. At the earlier stages of the search when no feasible solution has been found so far, the incumbent is taken to be the *least infeasible* solution (the one with smallest value $s(.)$) known so far.

Additionally, at some steps of the Intensification Phase we consider swap interchanges within the current routes. That is, we consider each route independently and we perform a sequence of interchanges of two arcs within the route until we obtain a 2-opt route. In what follows this will be referred to as exploring $N_{2-opt}(x)$.

Initially, $\hat{x}$ denotes the solution obtained with the rounding procedure. At the different steps of the Intensification Phase, $\hat{x}$ denotes the current solution. The Intensification Phase is as follows:

- Explore $N_{2-opt}(\hat{x})$
- While termination criteria do not hold do:

  IPh_1 : Select the best non-tabu move in $N1(\hat{x}) \cup N2(\hat{x})$ according to objective function (17) (bypass tabu constraints if the aspiration criterion can be applied)

  IPh_2 : If the incumbent solution has been updated, explore $N_{2-opt}(\hat{x})$

An Intensification Phase terminates when i) the maximum number of iterations has been reached, or ii) the maximum number of consecutive iterations without finding an acceptable move has been reached.

3.3 The Diversification Phase

The neighborhoods considered during the Intensification Phase may change the assignments of clients to open plants but, in all cases, the set of open plants remains fixed throughout that phase. However, for finding good quality solutions it may be necessary to consider different sets of open plants, in order to explore larger areas of the solutions space. The Diversification Phase performs moves that affect the set of open plants, so new solutions associated with different sets of open plants can be generated by the Intensification Phase The three neighborhoods that are explored in this phase are the following:

$N3$ **Close plant:**
For a given solution x, $N3(x)$ (see Figure 5) considers solutions y where the set of open plants differs from $O(x)$ exactly in one plant that is open in x and is closed in y. That is, $O(y) = O(x) \setminus \{i_1\}$ for some $i_1 \in O(x)$. Plants that are not tabu active are ordered in a list by increasing values of the ratio $used_cap/v_i$. The plant that is closed is the first one from the list that can be closed without violating the aggregated demand constraint. If such a plant does not exist, no move is performed. When a plant is closed its assigned customers are reassigned to the cheapest route (w.r.t. the original objective function) with the nearest-insert criterion. Customers are reassigned in the same order they had in the route of the closed plant.

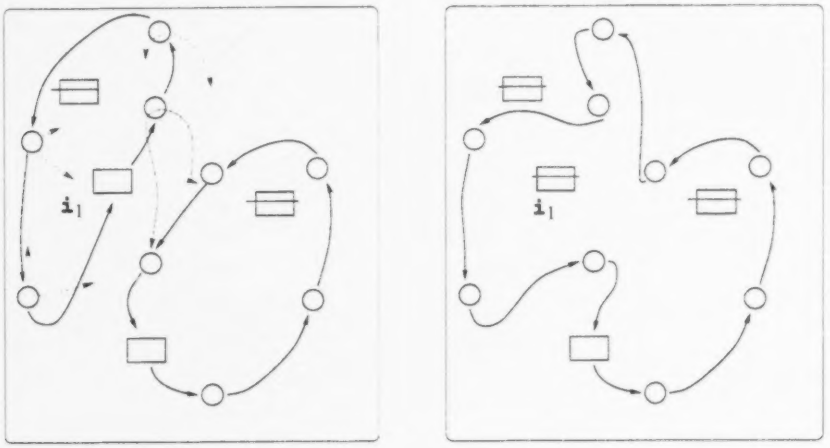


Figure 5: Neighborhood $N3(x)$. Close a plant and reassign its customers

$N4$ Switch plants:
For a given solution x, $N4(x)$ (see Figure 6) considers solutions y where the set of open plants differs from $O(x)$ exactly in one plant that is open in x and is closed in y, plus another plant that is open in y and is closed in x. That is, $O(y) = O(x) \setminus \{i_1\} \cup \{i_2\}$ for some $i_1 \in O(x), i2 \notin O(x)$. We consider all non-tabu pairs (i_1, i_2), $i_1 \in O(x), i2 \notin O(x)$ such that: *a)* $O(x) \setminus \{i_1\} \cup \{i_2\}$ satisfies the aggregated demand constraint (11), and *b)* the total demand of clients assigned to i_i does not exceed the capacity of plant i_2. Condition

a) does not guarantee that the selected sets of plants are feasible. The reason is that a set of plants that satisfies (11) may not have any feasible assignment of clients. However, we use condition a) as a surrogate for the feasibility of the sets of open plants because the decision problem to know if a given set of plants has a feasible assignment is itself NP-complete. We select the admisible move with the best value with respect to the modified objective function (17). All clients assigned to i_1 in solution x, are reassigned to i_2 in solution y. The relative order of clients on the route of i_2 is the same as in the route of i_1, although the starting (and the ending) client of the route has to be decided. This is done by applying the nearest-insertion criterion to place plant i_2 on the route.

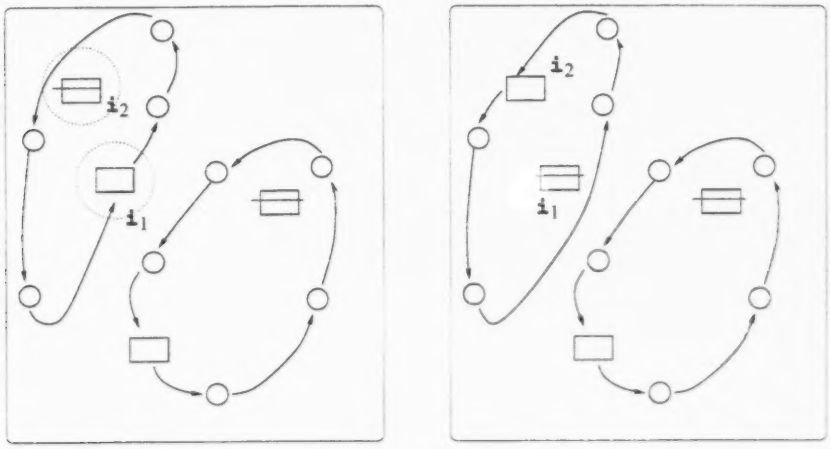


Figure 6: Neighborhood $N4(x)$. Switch open plant i_1 and closed plant i_2 by closing i_1 and opening i_2. Assign to i_2 the route of i_1.

$N5$ **Open a plant:**
For a given solution x, $N5(x)$ (see Figure 7) considers solutions y where the set of open plants differs from $O(x)$ exactly in one plant that is closed in x and is open in y. That is, $O(y) = O(x) \cup \{i_1\}$ for some $i_1 \notin O(x)$. The plant that is opened is the one with the smallest value $\frac{f_i}{v_i}$ among the non-tabu plants. When a plant i_1 is opened, some customers from other routes are assigned to i_1 as follows: Customers j are considered in turn by increasing values of $c_{i_1 j}$. While the capacity of plant i_1 is not violated clients are reassigned to i_1, using the nearest-insert criterion to place clients on the route of i_1.

After performing a move in any of the three neighborhoods, the neighborhood N_{2-opt} is explored to improve the routes. Also, if a plant has been opened or closed it remains tabu active (cannot be closed if it has been opened, or cannot be opened if it has been closed) for a fixed number of Diversification Phases. In the case of $N4$, the aspiration criterion is used to allow tabu moves that improve the incumbent solution. The order in which the neighborhoods are explored is as follows. $N3$ is first explored. If no admisible move in $N3$ is found, $N4$ is explored. $N5$ is only explored when no admisible move has been found neither in $N3$ nor in $N4$. If no feasible solutions are known for the current set of open plants at the beginning of the Diversification Phase, $N3$ is not explored and the Diversification Phase starts by exploring $N4$.

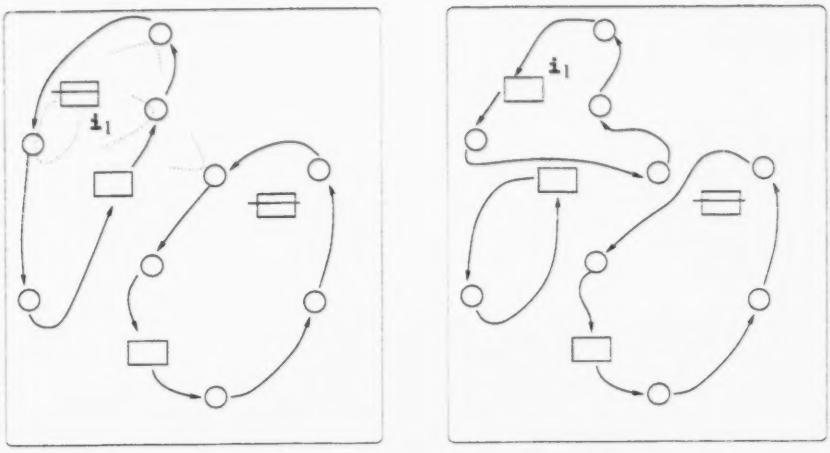


Figure 7: Neighborhood $N5(x)$. Open a plant, and assign some customers to it.

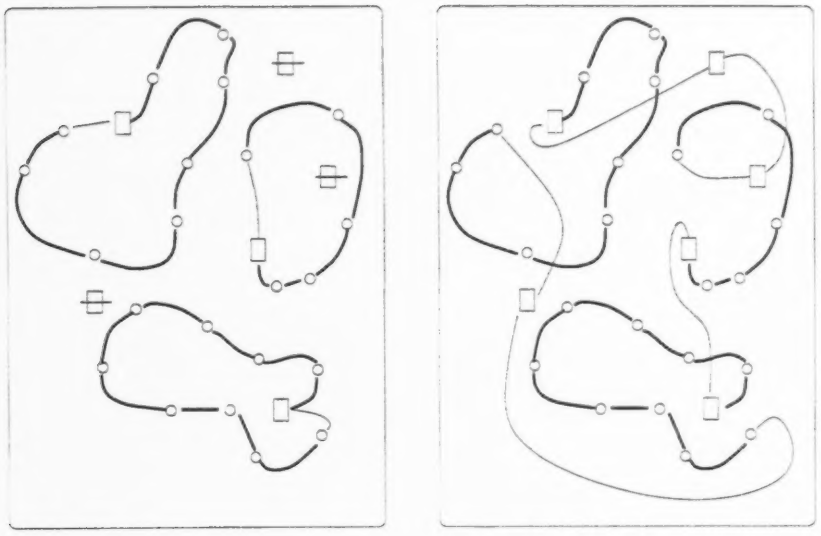
4 Lower bound

We now describe a lower bound for the considered LR problem that is not derived from the model LR1. In the Computational Experiments Section this bound will be compared with the lower bound obtained with the LP relaxation of LR1 and with the upper bounds obtained with the Tabu Search heuristic of the previous section. The bound consists of two terms. The first one is derived from the costs of the edges that connect clients among them, as well as the costs for starting the routes (connecting plants with clients). The second term is derived from the costs incurred when opening the plants, as well as the costs for terminating the associated routes (connecting clients with plants).

In order to derive the first term we consider an Asymmetric Traveling Salesman Problem ($ATSP$) defined on a complete digraph K_{m+n} where the set of nodes is given by $I \cup J$. That is, there is one node associated with each site for a possible location as well as one node associated with each client. We define the following cost function on the arcs of K_{m+n}:

$$\hat{c}_{ij} = \begin{cases} c_{ij} & \text{if } i \in I, j \in J \\ c_{ij} & \text{if } i, j \in J \\ 0 & \text{otherwise .} \end{cases}$$

The cost function for $ATSP$, $\hat{c}$, takes into account the original costs c only for i) the arcs that connect plants with clients, and ii) the arcs that connect clients among them. The $\hat{c}$-costs relative of the arcs that connect clients with plants and plants among them are defined to be 0. Let $ATSPb$ denote the value of the optimal solution to the $ATSP$ defined on K_{m+n}. As depicted in Figure 8 any feasible set of routes to the considered LR problem can be transformed into a feasible solution to the $ATSP$ defined on K_{m+n}, after arbitrarily assigning a direction to the routes of the LR solution.



a) Solution to the LRP

b) Derived solution for the ATSP

Figure 8: Feasible solution to the ATSP

The $\hat{c}$-cost of this transformed solution is equal to the c-cost of the arcs of the routes that connect plants with clients, plus the c-cost of the arcs of the routes that connect clients among them, since those are the only non zero $\hat{c}$-costs. Note that the $\hat{c}$-cost of this transformed solution does not take into account the c-cost of the last stage of the route. That is, it does not consider the cost of connecting the last client of a route with the plant. Therefore, $ATSPb$ is a valid lower bound on *"routing costs - costs for terminating the routes"* for any set of routes that visits all clients.

We next obtain the term derived from the cost for opening the plants and for terminating the routes. For $i \in I$, if plant i were to be open, we would certainly incur on a cost f_i but, since some non-empty route should be associated with i, we would also incur on the cost for terminating its associated route. Let $j_1(i) \in \arg\min_{j \in J} c_{ij}$ denote the index of the closest client to plant i. Therefore, $\tilde{f}_i = f_i + c_{ij_1(i)}$ is a lower bound on the cost for opening plant i and terminating its associated route. Let KPb be the optimal value to the following Knapsack Problem:

$$(\text{KPb}) = \min \sum_{i \in I} \tilde{f}_i y_i \tag{18}$$

$$\sum_{i \in I} v_i y_i \geqslant D \tag{19}$$

$$y_i \in \{0,1\}, \ i \in I. \tag{20}$$

Note that the optimal solution to (18)-(20) would be the same if the right hand side of constraint (19) were LC.

It is clear that $LB = ATSPb + KPb$ is a valid lower bound on the value of LR1.

In fact, the term KPb can be strengthened by taking into account that possibly not all the capacity of a given plant can be actually consumed by the clients' demands. In particular, the *profitable* capacity of a plant $i \in I$, $\bar{v}_i$, is the maximum capacity of the plant that can be consumed by the clients demands. It is given by:

$$\begin{aligned} \bar{v}_i = \max & \sum_{j \in J} d_j z_j \\ & \sum_{j \in J} d_j z_j \leqslant v_i \\ & z_j \in \{0,1\},\ j \in J. \end{aligned}$$

Now, KPb can be strengthened to $\overline{KPb}$ obtained by solving (18)-(20) after substituting the coefficients v_i by $\bar{v}_i$.

5 Computational Experiments

In order to evaluate the quality of the proposed lower and upper bounds we have performed a series of computational experiments. To the best of our knowledge there are not available benchmark instances for the location\routing problem described in this work. For this reason we have obtained a set of 125 test instances from the data sets for to the Two-Stage Capacitated Facility Location\Routing instances of

`http://troubadix.unisg.ch/klose/problems/problems.html\#TSCFLP` .

Problems are classified in five groups according to their dimensions. Each group consists of twenty five instances. Groups $S1$, $S2$ and $S3$ contain instances with 5 plants and, respectively, 10, 20 and 30 clients. Groups $M2$ and $M3$ contain instances with 10 plants and, respectively, 20 and 30 clients. Within each group, we have classified problems according to the ratio $r = TotalCapacity/AgregatedDemand$. The possible values for this ratio are $r = 4.5, 4, 2.5, 2$ and 1.5. Therefore, within each group, there are five subgroups (labeled with a, b, c, d and e) each of which contains five instances with ratio $r = 4.5, 4, 2.5, 2$ and 1.5, respectively.

Since the instances of the above URL correspond to the Two-Stage Capacitated Facility Locatio\nRouting Problems, we had to adapt the existing instances to instances for the Location\Routing problem that we study in this work. To this end, we have proceeded in the following way. First, we have only considered one stage. That is, the plants and clients of our instances correspond, respectively, to plants and depots of the existing instances. Second, when the existing instance had more plants and/or clients than we needed, we have ignored the data of the exceeding plants and/or clients. Finally, we evaluated the ratio $\hat{r} = TotalCapacity/AgregatedDemand$ for each instance, and the demands of the clients were scaled by the factor $r/\hat{r}$, so the resulting instance had a fixed ratio $TotalCapacity/AgregatedDemand$ equal to r.

All the algorithms are coded in C language. Some routines of the CPLEX 6.5 callable library

(ILOG 1999) are used in the program that solves the LP relaxation $RLR1$. The exact solution to the Asymmetric Traveling Salesman Problem that is solved to obtain the lower bound presented in Section 4, is obtained with the Toth and Carpaneto (1995) code for the asymmetric TSP. The knapsack problem that has to be solved to complete the above lower bound is solved with the Martello and Toth (1990) code. This code is also used to obtain the right hand side for constraint (11) in the LP relaxation $RLR1$. The experiments have been performed on a SUN sparc station 10/30 using one of its 4 hyperSPARC processors at 100 MHz., SPECint95 2.35. After some tuning the following values for the parameters were used in the Tabu Search heuristic:

- The intensification phase terminates after 2000 iterations or after 1000 consecutive iterations without finding any feasible solution.
- The tabu tenure for a tabu move is 5 iterations, both in the intensification and the diversification phases.
- The overall heuristic terminates after $|I|$ diversification phases or if during an intensification phase no feasible move was found.
- For updating the penalty term $P := P \times \alpha^{\beta}$ in the objective function, α is initially set to the value 2 and updated every $niter2 = 100$ iterations according to $\alpha := min\{\alpha + 0.01, 3\}$. The value of β is initially set to 1 and updated every $niter1 = 10$ iterations according to $\beta = (infeas/(niter1 - 1)) - 1$, where $infeas$ is the number of infeasible solutions obtained in the last $niter1$ iterations.

We have used CPLEX 6.5 to solve exactly the small instances. The value of the optimal solution provides us with a reference to compare our results, both the lower and the upper bounds. Despite their small size, the instances in these groups were very difficult to solve exactly for CPLEX 6.5, especially those in subgroups with smaller value of r (tighter ratio capacity/overall-demand). For this reason we added some termination criterion to stop the search for the optimal solution when optimality could not be proved within some pre-specified limits. Using this termination rule we were able to optimally solve all problems in group $S1$, all problems in subgroups $S2a$ and $S2b$, and three instances in subgroup $S2c$. We could not find the optimal solution for the remaining 12 instances. In particular, the termination criteria used was to stop the procedure after 48 hours of running time.

Tables 1- 3 depict a summary of the results obtained with our test instances. The first two columns in Table 2, $plants \times clients$ and r respectively, give the dimensions of the instances and the value of the parameter r. Table 1 gives the deviations from the optimal solution for the small instances solved with CPLEX 6.5. For the instances where the optimal solution could not be found, the deviations have been evaluated relative to the valid lower bound provided at termination by CPLEX. In Columns 1-12, each group of three columns under the same heading gives the average value and the range (minimum and maximum value) of the percent deviation of our lower or upper bounds from the optimal/lower-bound solution obtained with CPLEX ($best$). Columns 1-3 (dev $lb1$) give the results of the percent deviation $(100(best - lb1)/best)$ of the lower bound obtained with the LP relaxation $RLR1$ ($lb1$). Columns 4-6 (dev $lb2$) give the values of the percent deviation $(100 * (best - lb2)/best)$ of the lower bound derived in Section 4 ($lb2$) (this bound has been calculated without reinforcing the term KPb). The next three

columns, Columns 7-9, give the results of the rounding heuristic. In particular, Columns under *dev ub*1 give the percent deviation $(100 * (ub1 - best)/best)$ of the upper bound obtained with the rounding heuristic ($ub1$). The results of the Tabu Search heuristic are presented in Columns 10-12 (*dev ub*2) that depict the values of the percent deviation $(100 * (ub2 - best)/best)$ of the upper bound obtained with the Tabu Search heuristic $ub2$. Finally, columns under *#optima* give the number of instances of each subgroup optimally solved by CPLEX and with our Tabu Search heuristic ($ub2$), respectively.

As can be seen, the lower bounds derived from $RLR1$ are quite weak, since the average percent deviation from the optimal is nearly always above 10 percent. On the contrary, the lower bounds $lb2$ derived in Section 4 improve notably on the values of $lb1$ and give an average percent deviation from the optimal values which never exceeds 1%, excepting for Subgroup $S2e$ for which the average deviation is close to 2%. This bad average is due to the effect of one specific instance where the deviation is as big as 10%. For this instance the term KPb of $lb2$ corresponds to a set of plants with an overall capacity that is exactly equal to the overall demand. We have checked that this set of plants is not feasible (there is no feasible assignment of clients within it). However, if for this instance we calculate $lb2$ with the reinforced term $\overline{KPb}$ the obtained percent deviation reduces to less than 1%. It should also be noted that there is one instance in Subgroup $S2e$ for which our lower bound $lb2$ is 2% better than the lower bound provided by CPLEX at termination. Despite the simplicity of the rounding heuristic, for all the instances in $S1$ and $S2$ a feasible solution was obtained. However, as could be expected, the upper bounds obtained with the rounding heuristic are not very good and some of the average deviations *dev ub*1 are above 20%. This is because the rounding heuristic tends to open plants with low ratio v_i/b_i even if their capacity is larger than necessary. On the contrary, the results obtained with the Tabu Search heuristic are very satisfactory, since 34 out of the 38 instances optimally solved by CPLEX, were also optimally solved with our heuristic. The small values of the deviations *dev ub*2 show that for the instances that were not optimally solved the quality of the obtained solution is always very good.

To a great extent, the results of Table 1 confirm the results of Table 2 where we show percent gaps between the upper and lower bounds. In Columns $3-8$ each group of three columns under the same heading gives the average value and the range (minimum and maximum value) of the percent deviation of the upper bound from the lower bound. In particular, Columns $3-5$ $(gap\ ub1 - lb1)$ give the percent deviation $(100(ub1 - lb1)/lb1)$ of the upper bound obtained with the rounding heuristic ($ub1$) from the lower bound obtained with the LP relaxation $RLR1$ ($lb1$). Similarly, Columns $6-8$ $(gap\ ub2 - lb2)$ give the percent deviation $(100 * (ub2 - lb2)/lb2)$ of the upper bound obtained with the Tabu Search heuristic ($ub2$) from the lower bound of Section 4 ($lb2$). For problems in groups $S1$ and $S2$ the reader can check that the values of Columns 3 and 6 of Table 2 are indeed an upper bound of the sum of the individual deviations of the lower and the upper bounds from the optimal solution, shown in Columns 1 and 7 of Table 1 for $gap\ ub1 - lb1$, and in Columns 4 and 10 of Table 1 for $gap\ ub2 - lb2$. For the instances in Groups $S3$, $M2$, and $M3$ the obtained results follow the tendency of the smaller instances. More specifically, the rounding heuristic succeeded in finding a feasible solution in 118 out of the 125 instances although, in general the quality of the upper bound is quite poor.

The values of $gap\ ub1 - lb1$ show again the weakness of both the lower bound $lb1$ and the upper bound $ub1$, whereas the values of $gap\ ub2 - lb2$ are, in general, small confirming the good

results obtained for the small instances with the lower bound $lb2$ and the upper bound $ub2$. Figure 9 gives an example of the evolution of *gap* $ub2 - lb2$ in the Tabu Search heuristic for one instance of dimension 10*20 that was selected arbitrarily for illustration. The thin dotted lines represent the times when diversification steps took place whereas the thick break points represent the times where the upper bound $ub2$ was updated and the actual values of *gap* $ub2 - lb2$.

More insight about the quality of $ub2$ and $lb2$ can be obtained from the last columns of Table 2 that depict the number of instances of each subgroup for which *gap* $ub2$ is between some fixed threshold. It can be observed that for 98 out of the 125 instances the gap between $ub2$ and $lb2$ is below 1%. For 24 instances this percent gap is in $[1, 1.25)$, for 2 instances it is in $[1.25, 5)$, and only for 1 instance this gap is above 5%.

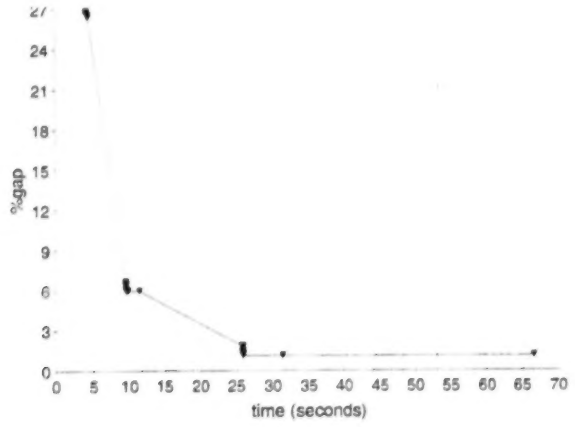


Figure 9: Evolution of $ub2$ on problem 3M2b

We have further investigated the instances for which the *gap* $ub2$ is big. In one case, the gap reduces from 13.17% to 0.67% when the lower bound $lb2$ is calculated with the strengthened term $\overline{KPb}$. In the remaining cases, the bad quality of the upper bound $ub2$ causes this big percent gap. Values of $ub2$ far from the optimum seem to occur when the tabu search procedure does not succeed in finding the optimal set of plants to open.

The times required to obtain the lower bounds $lb1$ and $lb2$, and the upper bound $ub2$ are depicted in in Columns 1-9 of Table 3. Again each group of three columns under the same heading gives the average values and the ranges (minimum and maximum value). We have not decomposed *time* $lb2$ in the times required by each of the two procedures used to obtain $lb2$ (the time required to solve the Traveling Salesman Problem, and the time required to solve the Knapsack problem) because the time for solving the Knapsack Problem is negligible with respect to the time for solving the Traveling Salesman Problem. Note that the lower bound $lb2$ is obtained with a small computational effort, since the values in *time* $lb2$ are quite small and considerably smaller than the corresponding values in *time* $lb1$. As to the times required to obtain the upper bound $ub2$, again we can conclude that these times are good taking into account i) the difficulty for optimally solving the instances with CPLEX 6.5, and ii) the quality of the bounds obtained with the Tabu Search heuristic. Columns 10-12 of Table 3, show the times

required by the Tabu Search heuristic to obtain the best solution found. This information is also depicted in Figure 10, where each stack bar corresponds to one test instance and represents the total time spent by the Tabu search heuristic on the instance. The dark part of each bar shows the time required to obtain the best solution found, so the light part of each bar shows the time spend since the best solution was found until termination. As can be seen, the times to obtain the best solutions are considerably smaller than the total times. Thus, we could have adjusted the termination criteria of the heuristic to obtain similar results in smaller computation times.

As usual, we can observe an increase in the computational times as the size of the problems grows. We have not performed a rigorous analysis but the times required to solve RL1 seem to increase approximately quadratically with $|I| + |J|$. For the Tabu Search heuristic, the increase of times seems to be approximately quadratic with $|I| \times |J|$. As to the times to obtain $lb2$, they are very small. It is surprising that for many problems in the group M computation times are smaller than for other instances in group S, even with the same number of customers. We attribute this to the fact that a code for the Asymmetric Traveling Salesman Problem is used. Note that the only source of asymmetry in the ATSPs that are solved in this work is related to plants. Thus, problems in group S are much more symmetric than problems in group M.

As expected, we have not appreciated any influence of the value of the parameter r on the difficulty for solving RL1. For the lower bound $lb2$ we can appreciate a slight increase on the computational effort as the value of r decreases (for problems of the same size). In the case of the Tabu Search heuristic, the influence of the value of r on the required times is smaller to what we thought it would be when designing the computational experiments. Nevertheless, in general terms, problems with intermediate values of r seem to require higher times.

Finally, Columns 13-19 in Table 3 give information about the structure of the best solutions found with the Tabu Search heuristic. In particular, Columns % *used cap* show the percentage of overall the capacity of the set of open plants that is consumed by the clients' demands, and Columns *plants open* show the number of open plants in the best solutions of the Tabu Search heuristic. We can see that the ranges for the values % *used cap* are, in general very high and that in some cases, the maximum value of a subgroup is 100%, which means that the procedure tends to open as few plants as possible. To a large extent this can be explained by the fact that in the considered instances the opening costs are much larger than routing costs. We should expect that if the opening costs were larger, the number of open plants would increase and the percent of consumed capacity would tend to decrease.

6 Conclusions

In this work we propose a new model for a combined Location\Routing problem and we obtain tight bounds for the problem. Upper bounds are obtained with a Tabu Search heuristic. The first phase of the heuristic is a rounding procedure that is applied to the solution of the LP relaxation of the model. We also propose a lower bound that consists of two terms that are derived from the set-up costs and the routing costs, respectively. In general, this bound improves considerably on the LP lower bound. Further research will focus on exploiting the structure of the model as to apply the column generation methodology, and to extend the model to other more general Location\Routing problems.

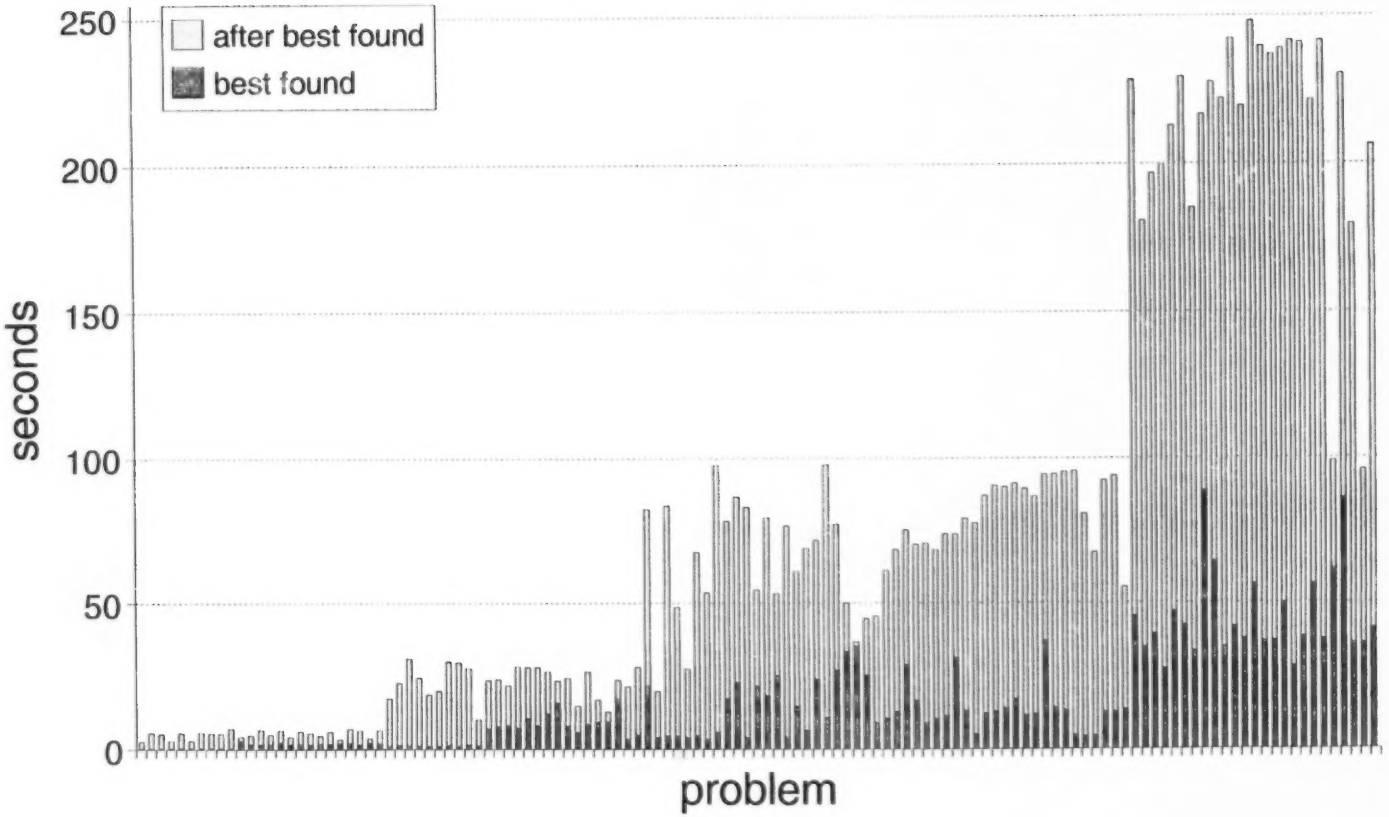


Figure 10: Tabu Search:Time to find the best solution vs. total cpu time

Table 1: Percent deviation from optimal/lower-bound for small instances

	dev lb1			dev lb2			dev ub1			dev ub2			# optima	
	avg.	min	max	avg.	min	max	avg.	min	max	avg.	min	max	CPLEX	ub2
S1a	14.15	4.61	23.19	0.88	0.60	1.07	12.35	3.86	19.03	0.00	0.00	0.00	5	5/5
S1b	11.10	5.34	18.79	0.89	0.64	1.07	8.56	0.00	16.36	0.00	0.00	0.00	5	5/5
S1c	20.14	17.64	23.50	0.71	0.45	0.94	20.82	12.50	30.13	0.00	0.00	0.00	5	5/5
S1d	11.59	4.09	21.77	0.69	0.52	0.94	3.77	1.28	5.75	0.00	0.00	0.00	5	4/5
S1e	10.79	7.30	13.80	0.56	0.30	0.80	11.46	7.42	18.24	0.03	0.00	0.13	5	4/5
S2a	15.29	0.68	26.23	0.69	0.54	1.03	9.86	0.00	26.66	0.00	0.00	0.00	5	5/5
S2b	13.93	0.68	29.70	0.67	0.54	0.94	2.48	0.00	7.24	0.00	0.00	0.00	5	5/5
S2c	16.47	13.82	18.81	0.46	0.36	0.65	21.42	5.53	38.56	0.02	0.00	0.05	3	1/3
S2d	9.44	5.42	13.68	0.45	0.38	0.51	8.85	0.78	14.64	0.23	0.06	0.45	0	
S2e	7.40	0.00	19.73	1.88	-2.10	10.30	6.85	0.37	10.48	0.88	0.02	2.91	0	

Table 2: Percent gaps between upper and lower bounds for all instances

	plants × clients	r	gap ub1-lb1			gap ub2-lb2			distribution gap ub2			
			average	min	max	average	min	max	< 1%	[1%,1.25%)	[1.25%,5%)	> 5%
S1a	5 × 10	4.5	31.99	8.88	48.50	0.99	0.72	1.15	1	4	0	0
S1b		4	22.62	9.72	41.50	0.98	0.64	1.15	1	4	0	0
S1c		2.5	51.53	36.60	70.10	0.86	0.54	1.15	3	2	0	0
S1d		2	17.82	9.02	29.45	0.73	0.53	0.99	5	0	0	0
S1e		1.5	25.12	16.80	37.17	0.65	0.33	1.04	4	1	0	0
S2a	5 × 20	4.5	31.64	0.69	59.47	0.78	0.56	1.15	4	1	0	0
S2b		4	20.90	0.69	42.24	0.69	0.56	0.94	5	0	0	0
S2c		2.5	45.74	22.45	70.66	0.58	0.42	0.77	5	0	0	0
S2d		2	25.27	6.55	43.83	0.66	0.52	0.77	5	0	0	0
S2e		1.5	16.33	1.11	36.37	3.24	0.62	13.17	4	0	0	1
S3a	5 × 30	4.5	43.64	1.91	102.85	1.01	0.78	1.13	2	3	0	0
S3b		4	50.89	18.68	99.75	0.96	0.66	1.13	2	3	0	0
S3c		2.5	31.27	9.57	60.84	0.61	0.53	0.68	5	0	0	0
S3d		2	27.51	6.45	54.39	0.59	0.44	0.71	5	0	0	0
S3e		1.5	14.27	2.57	32.05	0.48	0.41	0.61	5	0	0	0
M2a	10 × 20	4.5	43.62	13.06	76.68	0.91	0.8	1.04	4	1	0	0
M2b		4	36.76	20.05	59.76	0.94	0.82	1.11	3	2	0	0
M2c		2.5	24.46	13.98	42.16	1.25	0.58	2.78	3	1	1	0
M2d		2	13.43	2.15	22.45	0.8	0.62	1.15	4	1	0	0
M2e		1.5	10.89	10.89	10.89	0.84	0.7	0.9	5	0	0	0
M3a	10 × 30	4.5	38.64	10.11	69.52	0.74	0.63	0.82	5	0	0	0
M3b		4	37.91	16.71	56.90	1.28	0.62	3.34	4	0	1	0
M3c		2.5	21.39	5.60	32.12	0.78	0.56	0.93	5	0	0	0
M3d		2	12.05	5.06	24.04	0.57	0.47	0.69	5	0	0	0
M3e		1.5	9.81	7.02	12.71	0.63	0.38	1.02	4	1	0	0

Table 3: cpu times and solutions' characteristics

	time lb1			time lb2			time ub2			time best found			% used cap			plants open		
	average	min	max	average	min	max	average	min	max	average	min	max	average	min	max	average	min	max
S1a	0.28	0.26	0.30	0.02	0.01	0.03	4.46	2.70	5.68	0.28	0.26	0.29	85.49	78.77	92.67	1	1	1
S1b	0.27	0.23	0.29	0.03	0.02	0.04	5.30	2.90	7.02	0.27	0.22	0.30	88.45	73.54	99.57	1	1	1
S1c	0.27	0.23	0.32	0.02	0.01	0.03	5.42	4.23	6.64	2.00	1.48	2.90	95.79	91.19	100	2	2	2
S1d	0.28	0.24	0.31	0.02	0.00	0.03	5.28	4.23	6.11	1.62	1.41	1.82	94.62	84.15	98.48	2	2	2
S1e	0.25	0.24	0.26	0.02	0.00	0.04	5.43	3.42	7.02	1.89	1.65	2.04	95.86	92.75	98.58	3	3	3
S2a	1.26	1.18	1.37	0.09	0.02	0.18	22.90	17.42	31.17	1.20	1.08	1.37	84.59	68.79	94.85	1	1	1
S2b	1.22	1.11	1.37	0.09	0.02	0.16	23.54	10.23	30.00	1.20	1.06	1.38	80.32	63.50	92.71	1	1	1
S2c	1.25	1.10	1.42	0.10	0.04	0.17	25.13	21.77	28.38	8.11	7.03	10.50	93.19	85.58	99.37	2	2	2
S2d	1.30	1.11	1.43	0.14	0.06	0.25	23.43	14.69	28.12	9.95	5.74	15.77	96.51	94.56	99.11	2.2	2	3
S2e	1.34	1.23	1.42	0.14	0.03	0.28	20.23	12.74	26.52	9.63	3.48	17.22	96.75	88.42	100	2.8	2	3
S3a	4.53	3.88	4.95	0.24	0.12	0.48	52.55	19.79	83.72	7.93	3.95	21.89	83.47	66.17	95.64	1.2	1	2
S3b	4.77	3.42	5.88	0.24	0.13	0.51	64.91	27.65	97.66	7.06	3.46	17.31	72.85	50.88	98.56	1	1	1
S3c	4.69	3.86	5.70	0.24	0.12	0.51	71.50	53.33	86.91	18.23	3.81	25.12	87.27	81.43	94.35	1.8	1	2
S3d	4.72	4.05	5.50	0.39	0.24	0.82	75.11	60.83	97.87	11.93	4.23	23.76	89.86	76.95	99.68	2	2	2
S3e	5.26	4.44	6.14	0.39	0.22	0.84	50.86	36.92	77.15	26.00	9.03	35.23	95.22	90.97	99.28	2.8	2	3
M2a	4.05	3.39	4.60	0.04	0.03	0.06	68.88	61.12	74.95	15.52	9.01	28.95	96.57	92.15	99.41	2	2	2
M2b	3.72	3.27	4.21	0.04	0.02	0.05	74.36	68.14	79.08	14.20	4.95	31.38	98.13	97.34	99.12	2	2	2
M2c	3.94	3.56	4.42	0.04	0.03	0.06	89.85	87.20	91.42	13.51	11.83	16.98	97.82	95.14	100	3.2	3	4
M2d	4.01	3.76	4.28	0.08	0.04	0.11	93.42	87.00	95.64	16.27	4.76	37.26	98.82	98.26	99.05	3.8	3	4
M2e	3.98	3.74	4.19	0.08	0.07	0.09	77.99	55.38	94.04	9.46	4.34	13.39	99.36	98.54	100	5	4	6
M3a	19.76	13.05	27.20	0.13	0.06	0.34	203.58	180.86	228.16	38.80	27.37	46.99	95.85	92.00	99.64	2	2	2
M3b	21.33	15.84	25.20	0.14	0.06	0.33	215.97	185.11	229.12	52.69	33.49	88.76	95.87	93.19	100	2	2	2
M3c	17.43	14.10	24.82	0.13	0.06	0.33	236.97	219.24	247.79	41.98	36.86	56.22	99.66	99.13	100	3	3	3
M3d	15.52	12.00	19.40	0.26	0.17	0.52	236.61	221.26	241.26	42.10	28.13	56.40	98.51	96.31	100	3.8	3	4
M3e	17.47	15.23	22.62	0.26	0.17	0.53	162.08	95.92	229.96	52.26	36.14	86.01	98.87	95.46	99.82	5.4	5	6

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